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THE PURE THEORY OF UNEMPLOYMENT INSURANCE:
Hoping For The Worst When Insurance is Available

by

Martin C. McGuire*

Frequently, a resource owner can sell some or all of his endowment at a fixed price in good times, but be cut off from his market entirely in bad times. A worker may choose his hours between leisure and labor at a fixed wage in good times, but be thrown out of work altogether in bad. A small country may sell as much of its exportable as it pleases at the world price in normal times, but be cut off entirely from imports/exports in time of war or emergency. Other examples could be multiplied.

Given this prospect, the worker, small country, etc. should probably want to take out insurance against the bad outcome, giving up some of its resource or earnings in good times in return for earnings indemnification under the bad contingency. Depending on price, how much insurance is it rational to purchase? One common benchmark price or exchange rate between contingencies is the "fair price," sometimes also called the "actuarially fair price," defined as the price at which expected benefits equal expected costs, or expected monetary gain is nil. **This paper demonstrates that when fair insurance is available, the rational resource owner if he is risk neutral or risk averse will necessarily buy so much insurance that he would prefer the "bad" contingency to occur.**

This result amounts to a special instance of the Arrow-Debreu (1963, 1959) theorem that to achieve Pareto efficiency the number of markets must

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equal or exceed the number of contingencies plus goods. It derives from the necessity that an optimizing consumer must trade off the benefits from an optimal commodity mix under alternative contingencies against unequal state utilities when markets are incomplete. The result depends in no way on arguments from moral hazard (Pauly 1969). Those arguments hinge on the incentive insurance may provide a subject to take less care thereby raising the chance of the bad outcome, which he has insured himself against, but benefiting from the savings in effort on self protection. The result does depend qualitatively on an extended definition of Arrow-Pratt (1363, 1964) risk aversion as proposed by Kihlstrom and Mirman (1974) viz. a concave transformation of any one von Neumann-Morgenstern utility function representing the same ordinal preferences. But it does not involve any presumed ability to protect oneself against unemployment by probability improving "self-protection" measures (Erlich and Becker, 1972)) nor interactions between these and risk aversion (McGuire, Pratt and, Zeckhauser, 1991). On the contrary, in the case to be studied here the probability of trade cutoff or unemployment is completely fixed.

ASSUMPTIONS:

a. There are two mutually exclusive states of the world: "peace," "trade," or "employment" is the good state designated by "1" and I will use the terms interchangeably to designate it: "war," "autarchy." or "unemployment" is the bad state, and is designated by "o."

b. An agent possesses a fixed resource endowment $\bar{x}$. In the good state any part x_1 of this endowment can be sold at a constant unit price w to obtain $wx_1 = y$, and the rest of $\bar{x}$ consumed. In the bad state, no trade is allowed, and the entire endowment must be consumed.

c. The agent has a state independent, strictly quasi-concave utility function in two arguments, his consumption of x (xl if employed, x^o if not) and his consumption of y (yl if employed and y^o if not). This utility function, V, is linear homogeneous in the arguments (x,y), or is any smooth continuous concave or convex transformation U = f(V) of the same with continuously increasing or decreasing first derivatives, f'(V). Each indifference curve, by strict quasi-concavity, has strictly a diminishing marginal rate of substitution (MRS).

d. He acts so as to maximize the expected value of a Von Neumann-Morgenstern expected composite utility W = $pU^1(\bullet) + (1-p)U^o(\bullet)$, where p the probability of employment and (l-p) that of unemployment are known and fixed.

e. He can protect himself against the loss of y-consumption if unemployed by paying out the premium $y_s^1 = (1-p)y_s^o/p$ when (or if) employed. In return he receives insurance benefit y_s^o if unemployed. Since $-(1-p)y_s^o + py_s^1 = 0$ this insurance is actuarially fair.

THE OPTIMAL PURCHASE OF INSURANCE:

Theorem: At the maximum of W = W*, $U^{o*} > U^{1*}$,(where "*" indicates solution values), provided $f'' \leq 0$ throughout. In words, to maximize his expected utility, this agent will purchase so much fair insurance that he is better off if the "bad" event is realized and he collects his insurance benefit.

Proof: The maximand for this problem is

(1) $$W = pU^1[(\bar{x}-x_1),(wx_1-(1-p)y_s^o)] + (1-p)U^o[\bar{x},py_s^o]$$

First order conditions with respect to x_1 and y_s^o are:

(2) $$\bullet\ U_x^1 + wU_y^1 = 0$$

(3) $$\text{-}\ U_y^1 + U_y^o = 0$$

where U_h^k indicates the marginal utility of good h in state k.

CASE I: U linear homogeneous, ie risk-neutral: $f(V) = U = V$

This is a pivotal case upon which the extensions to risk averse and risk preferring utility functions will hinge. It will be developed in the greater detail therefore. Since the marginal utility of good y is the same across contingent states the ratio of x to y consumed and the marginal utility of x must also be the same across contingencies. This follows from the homogeneity assumption. It follows from the budget-when-employed constraint that $x^{o*} > x^{1*}$. With $[x^{o*}/y^{o*}] = [x^{1*}/y^{1*}]$, it then also follows that $y^{o*} > y^{1*}$. With consumption of both commodities higher in war (unemployment) than in peace (employment) it follows that utility must be higher in the former state. QED.

This result is readily pictured in Figure 1. There are two parts to the diagram, that to the left of the vertical through the resource endowment, $\bar{x}$, and that to the right of this vertical. To the left of $\bar{x}$ is shown the opportunity set during employment and the effects on utility-when-employed of endowment sales. The x-endowment may be sold for two purposes: to obtain y for consumption when employed, or to pay insurance premiums for benefits if unemployed. To the right of $\bar{x}$ is shown the intercontingency transformation of premiums into insurance reimbursements at fair-odds prices. Under conditions of employment or peace, exchange at price w would be pursued along price line q^o __if no insurance were purchased__ up to a point of tangency at U^{1i} (i being initial utility when employed). If no insurance actually were purchased, unemployment would give consumption point $(\bar{x},0)$ and utility U^{oi}. However, by spending y_s^{1k} to purchase y_s^{ok} of insurance at the fair price of $y_s^{1k} = [(1-p)/p]y_s^{ok}$ the peacetime employment line shifts in to q^k, utility if employed drops to U^{1k} and utility if unemployed rises to U^{ok}. As y_s^o and y_s^1 increase the consumption point when employed retreats along the income expansion path, S. (This expansion path has slope "s" indicating the if-employed consumption ratio y^1/x^1.) As consumption-when-employed retreats along S, the consumption point if unemployed climbs along the vertical through $\bar{x}$. The optimum in expected utility is reached at the intersection of these two curves giving y^{0*} and unemployment utility of U^{0*}.

A complementary way to summarize this result is also shown in Figure 2. There the cost (measured in expected utility terms) of insurance purchase is the reduction in $V^1 = U^1$ weighted by p, while the benefit again in expected utility terms is the gain in $V^0 = U^0$, weighted by (l-p). With V a CRS function, the weighted cost is linear as consumption-when-employed retreats along path S and the weighted gain is diminishing as consumption-when-unemployed climbs the vertical through $\bar{x}$. If the elasticity of substitution σ were (a) infinite, or (b) zero and indifference curves, therefore, were (a) straight lines or (b) sharp cornered, the benefit curve in Fig 2 would (a) also be linear or (b) be linear and have a kink at the benefit limit where the income expansion path and vertical through x intersect. The argument above simply demonstrates that the expected utility maximum where probability weighted marginal benefits equal probability weighted marginal costs occurs to the right of the intersection or equality of the two total, unweighted utility components. At that intersection or equality, $V^0 = V^1$, whereas to the right of it, utility-if-unemployed exceeds utility-if-employed.

CASE II: U risk-averse: $f' > 0$, $f'' < 0$

To address the effect of risk aversion, the "underlying" linear homogeneous function, V, is not altered; the same ordinal rankings and indifference curves as in Fig. 1 apply, except now these are renumbered. The necessary condition shown in eq. (3) can be re-written

(3a) $\cdot\ f'(V^1)\, V^1_y(s^1) + f'(V^o)\, V^o_y(s^o) = 0$

With the primitive utility function, V, first degree homogeneous, its first partial derivatives are functions of the consumption ratios, s^k, only. Since this is important in the argument to follow the dependence is shown explicitly. To show that $U^{o*} > U^{1*}$ after optimal insurance has been purchased at fair prices, assume otherwise. The indifference curve through (x^{1*}, y^{1*}) being convex with negative slope intersects the vertical through $\bar{x}$, at a lesser value of s; and at a still lower value of s for a lower value of V^o ie. $s^{1*} > s^{o*}$. This entails $V^{1*}_Y < V^{o*}_Y$. Whence to maintain eq (3a), $f'(V^{1*}) > f'(V^{o*})$; this in turn entails $V^{o*} > V^{1*}$, a contradiction. Thus if $f' > 0$ and $f'' < 0$, U^{o*} is strictly greater than U^{1*}. QED

Figure 3 illustrates, adjusting Fig 2 to incorporate diminishing returns to scale, and using for the origin of f(V), the value of V which can be achieved equally in or irrespective of which state of the world occurs, ie the value of $V = V^1 = V^o$. As shown, the concave transformation of V changes expected marginal utility cost of insurance from a constant to an increasing function of the amount of insurance provided, and at the same time accelerates the decline of the marginal benefit function. The upshot of these two effects is that **risk aversion reduces the eauilibrim insurance purchase but not so much that unemulovment becomes less desirable than ewlovment.**

CASE III: U risk-preferring: $f' > 0$. $f'' > 0$

The potential for multiple optima and corner solutions inherent in the non-convexities introduced by risk preference show up pointedly in this analysis as well. Compared with the risk neutral outcome, a positive convex transformation f(V) can lead to two types of alternatives. Imagine starting from the risk-neutral optimum (with $U^{o*} > U^{1*}$ of course), and increasing f'' slightly from zero. One possible effect is for **more insurance to be purchased and the superiority of "war" over "peace' to increase.** A second possibility is that risk preference requires purchase of less fair insurance-- so much less that war or unemployment is not fully insured and remains the less desirable outcome.

In the first of these cases the unemployment consumption point moves up the vertical above the risk neutral y_s^{o*}. In this region $U^{o*} > U^{1*}$, $s^{o*} > s^{1*}$ and $U_Y^{o*} < U_Y^{1*}$. Consistency with eq. (3a) requires $f'^{o*} > f'^{1*}$ and therefore risk preference in the utility function, $f'' > 0$.

In the second of the above mentioned cases, the unemployed consumption point moves below the risk neutral y^{o*}. In this region $s^{o*} < s^{1*}$ and therefore $U_Y^{o*} > U_Y^{1*}$. Consistency with eq. (3a) requires $f'^{o*} < f'^{1*}$. Thus if f'' > 0 then $V^{o*} < V^{1*}$.

Both these cases are illustrated in Fig. 4 where the probability weighted underlying function of benefits and costs from the risk-neutral case has been subjected to a convex transformation through the point where $U^o = U^1$. This increase in f'' causes the weighted marginal utility cost to change from constant to declining, and the weighted marginal utility benefit to change shape as shown. The possibilities for multiple solutions are clear.

FURTHER ANALYSIS OF RISK NEUTRALITY:

Corollary: It follows from the construction of Fig 1 that under risk neutrality optimized (insurance protected) unemployment utility is unaffected by the probability of unemployment. For lower values of p -- ie lesser likelihood of peace, uninterrupted trade, or employment -- the optimized value U^{o*} remains unchanged with all the utility deficit absorbed by lower and lower U^{1*}.

Proof: This follows from the homogeneity assumption. With $U_y^{1*} = U_Y^{o*}$ as required the unemployment and employment consumption bundles must lie on the same ray through the origin. The unemployment consumption bundle however must lie on the vertical through $\bar{x}$. The intersection of these two lines is unique. QED.

Corollary: As the probability of unemployment or trade disruption increases from zero to unity, gross earnings (inclusive of insurance premiums) when employed rise monotonically; insurance premium payments rise monotonically and at a rate faster than the rise in earnings; and the proportion of gross earnings replaced declines monotonically. There is a critical probability of unemployment l-p **below which** the rational insurer replaces more than his gross earnings and above which the rational insurer replaces less than his gross earnings. The corresponding critical value of p is p = w/(w + s), where w = wage rate, or sales price of exports, and s = slope of income expansion path at wage-price w.

Proof: See Figure 5. The slope s is shown as the ratio of average propensities to consume y and x; ie $s = [\gamma/(1-\gamma)]$; Consider a probability of employment, $\hat{p}$, such that it is optimal to expend the amount $\hat{y}_s^{1*} = [(1-\hat{p})/\hat{p}]y_s^{o*}$ pictured. This shifts the budget or wage line down as shown. The optimum resource supply then becomes $[(\hat{y}_c^{1*}/w)+(\hat{y}_s^{1*}/w)]$, which yields a gross earnings when employed of $[\hat{y}_c^{1*} + \hat{y}_s^{1*}]$, to be divided between consumption when employed of $\hat{y}_c^{1*}$, and insurance premium when employed of $\hat{y}_s^{1*}$. For the case shown, $\hat{p}$ is not low enough to induce so much work when employed that earnings in that contingency match insurance if collected. Only when the optimal insurance premium reaches $[(1-\tilde{p})/\tilde{p}]y_s^{o*}$ does the level of gross earnings when employed rise to equal the amount of insurance purchased, [In the case of CES utility with elasticity σ, and "input intensity parameters" δ for good x and 1-S for good y, the critical value of (1-p)/p becomes $[\delta s[\exp(-1/\sigma)]-s[\exp((1-\sigma)/\sigma)]]$.

Corollary: For every value of [p,(l-p)], there is a value of s (the slope of the income expansion path S ie the optimal proportion of y- to x-consumption) which induces the risk neutral consumer insuring at fair odds, to allocate his **entire** earnings-if-employed to insurance premiums. Moreover, such an insurer may give up "almost" his entire endowment of $\bar{x}$ to earn y to buy premiums to cover his unemployment.

Proof: By inspection of Figure 1, for any p, or fair odds line, if s is great enough, y_s^{0*} will be great enough in turn to require a y_s^{1*} which absorbs $\bar{x}$ in its entirety.

Effect of Wage-Price on Insurance: If good x is normal one wage rate maximizes insurance demand. This is shown in Figure 6. as $w_{i\text{-}max}$.

Extension to "Unfair' Insurance: Fair insurance is a rarity. Insurance companies usually add a "loading factor" for numerous reasons such as to recover administrative costs, to yield a profit, or to anticipate moral hazard or adverse selection. To represent this effect, let the unit price of insurance be not (1-p)/p which is the "fair" price, but instead [((1-p)/p)r +g]. This adjusts the fair price by a fixed unit charge "g" and a "risk inflator, r." With this adjustment the necessary conditions for a expected utility maximum become

$$(4) \qquad - U_x^1 + wU_y^1 = 0$$

$$(5) \qquad (1-p)U_y^o - p[((1-p)/p)r + g]U_y^1 = 0$$

If "g" in (5) above is zero and only the risk-inflation factor enters, the change compared to fair insurance is slight. (a) The optimal purchase of insurance remains fixed independent of p, though at a lower amount than with fair insurance. (b) It no longer is possible a priori to assert that welfare when unemployed is necessarily greater than when employed; the outcome can tip either way depending on the ordinal utility function. However, if earnings net of the insurance premium are fully replaced or more it is clear welfare is higher in the unemployed state; and it is clear moreover that the higher the likelihood of unemployment the more probable will this be true. (c) Similarly, as in the no-load fair insurance case, when the probability of unemployment (l-p) rises, unemployment utility is protected completely (though at a lower level than in the fair insurance case) and all burden is allocated to the state when employed. (d) Similarly, maximum insurance is purchased at the wage which generates maximum y/x consumption when employed. This occurs at the same tangency point as shown in Figure 6, except the amount of insurance is diminished.

When the fixed loading factor "g" takes on a non zero value the analysis becomes less predictable. For now as the probability of unemployment increases, the amount of insurance purchased first declines and then rises again. This can be seen from differentiating eq (5) to obtain

$$(6) \qquad U^o_{yy} dy_s - gU^1_y[(1-2p)/(1-p)^2]dp = 0$$

for $p \gtrless 1/2$ it follows $dy_s/d(1-p) \lessgtr 0$. Thus at $p = 1/2$ the amount of insurance reaches a maximum.

IMPLICATIONS AND CONCLUSIONS:

This analysis is representative of a broad class of situations in which individuals or groups are at risk of being thrown back on their own resources and cut off from markets. The message of the analysis is that a tendency exists for individuals subject to such risks to rationally desire so much insurance that they hope "disaster" strikes. This tendency is attenuated for risk averse individuals or if insurance cannot be bought at a fair odds price, but it still exists. This phenomenon may explains in part why unemployment insurance is not generally available on a private basis, let alone at "fair" insurance rates; it interacts powerfully with moral hazard to produce simply too great a temptation to become unemployed. Nevertheless, in some cases a type of unemployment insurance close to this may be provided by the employer. Sometimes if one becomes "rift" and/or his position declared unnecessary and not to be refilled, retirement on an unreduced pension at age 50 is allowed. It is not unknown for people to at least hope they become "rift". Employers and insurance companies face a not dissimilar situation when they provide or allow the employee to purchase "disability" insurance. A minor disability might not be so bad if one's earnings are fully replaced. Typically in private disability insurance plans --which definitely do not provide "fair" insurance -- a strict limit is set on the proportion of regular earnings which can be replaced.

Another application of this analysis arises in trade among nations. An emergency trade disruption which leaves a country's productive resources over-specialized in its export good is very similar to the worker unemployment case (McGuire, 1990). In fact the prevalence of other protective measures over insurance compacts with other countries may be explained by the adverse incentives inherent in "unemployment of resource insurance" as elaborated here.

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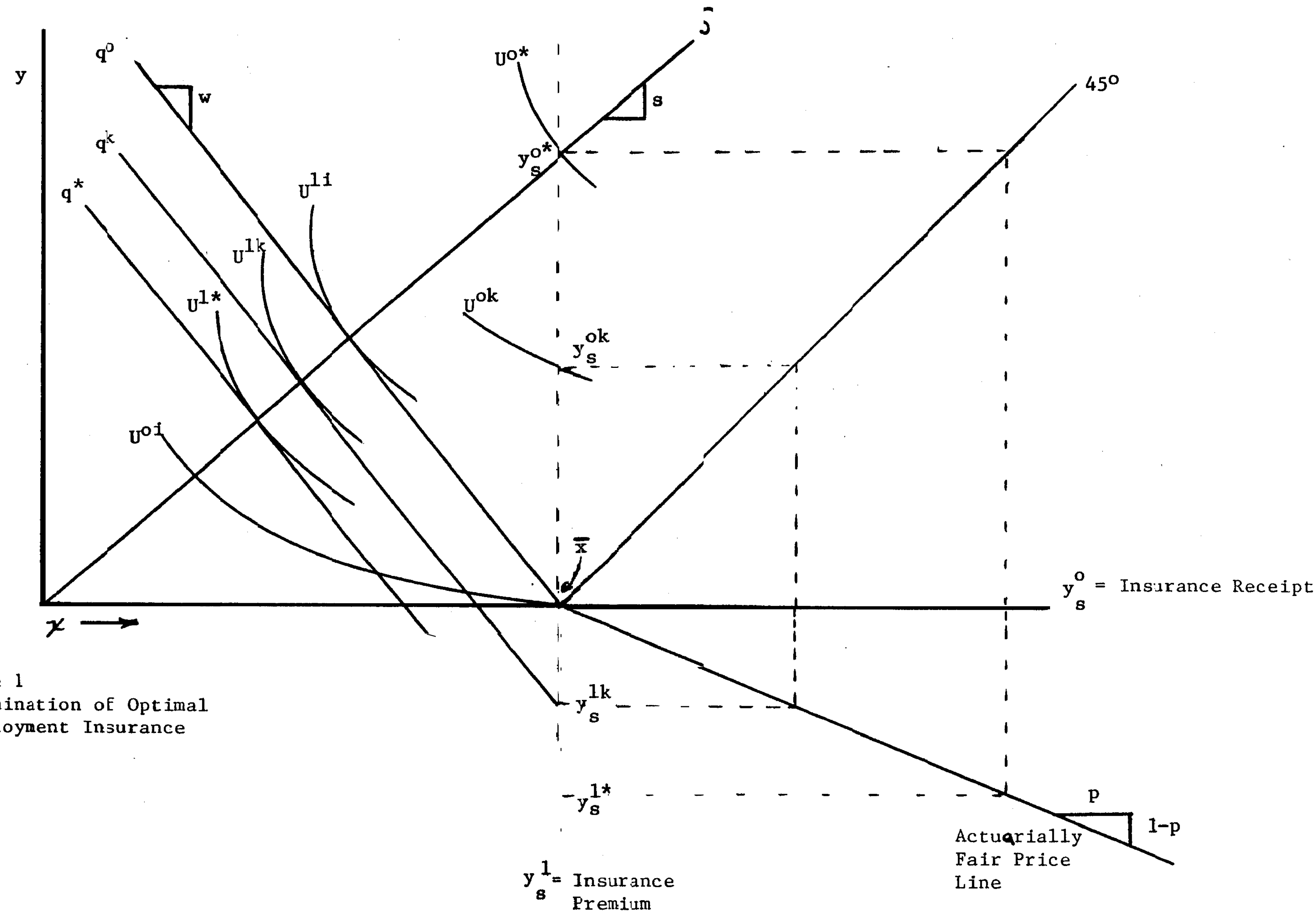


Figure 1
Determination of Optimal
Unemployment Insurance

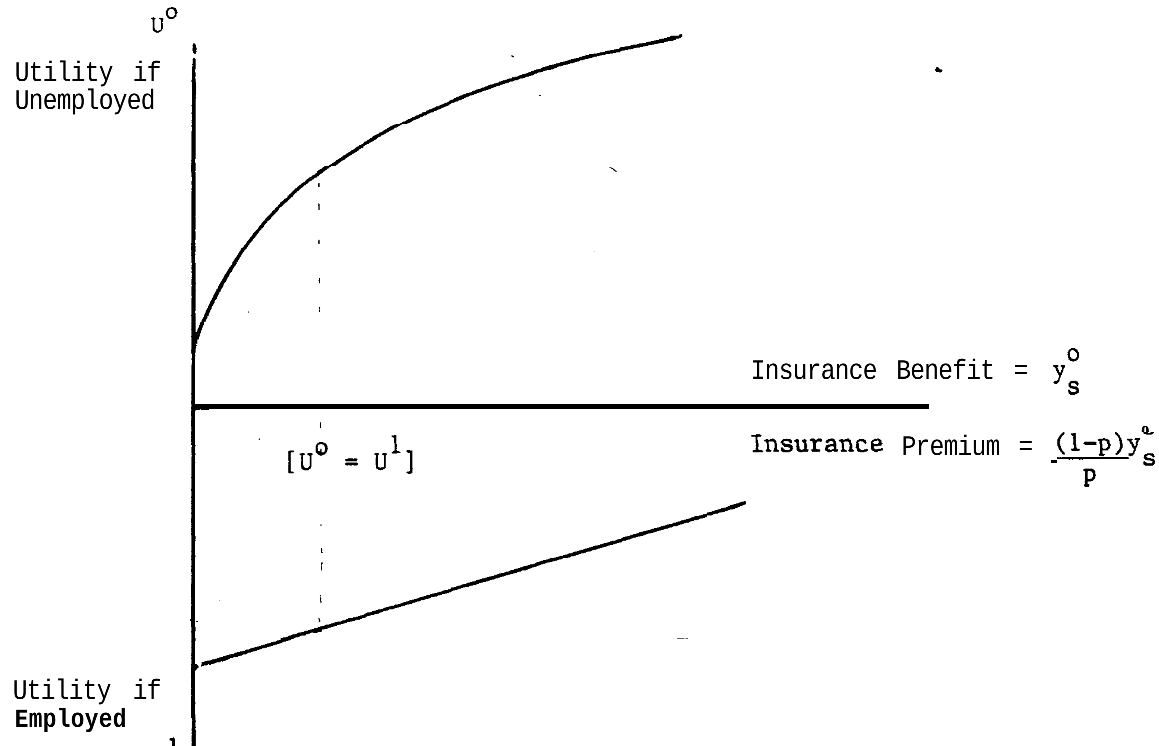
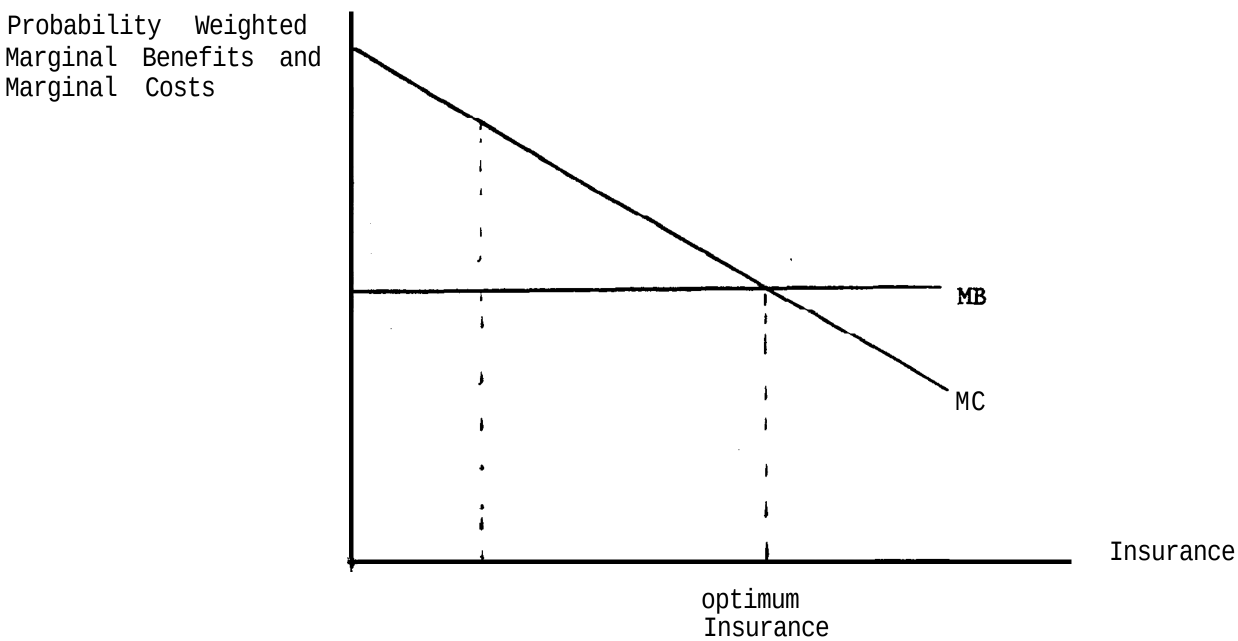


Figure 2

RISK NEUTRAL UTILITY



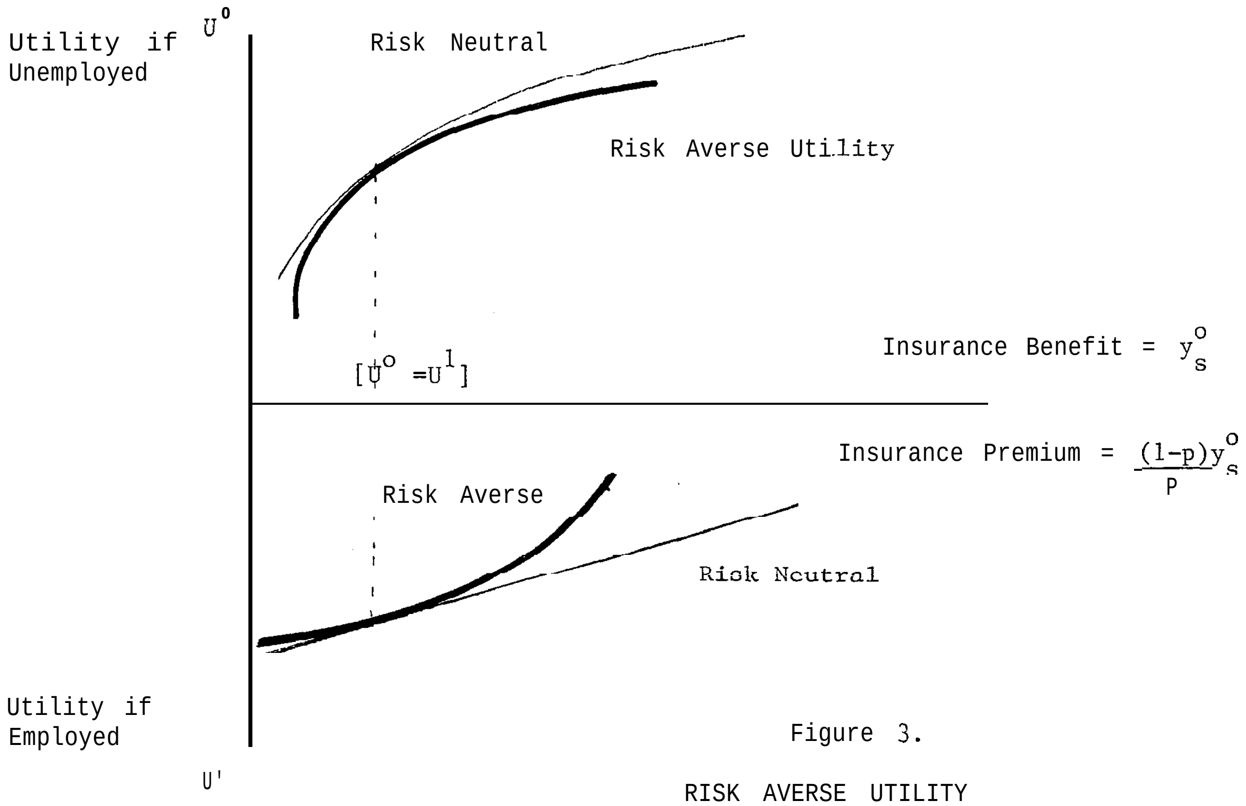
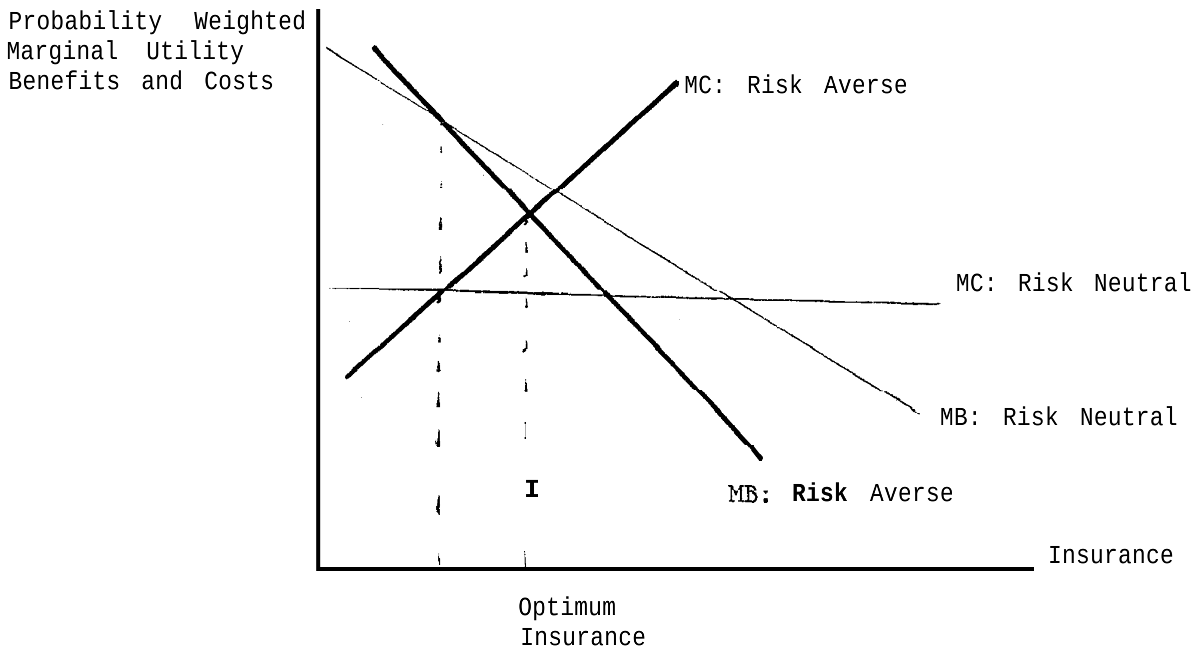


Figure 3.

RISK AVERSE UTILITY



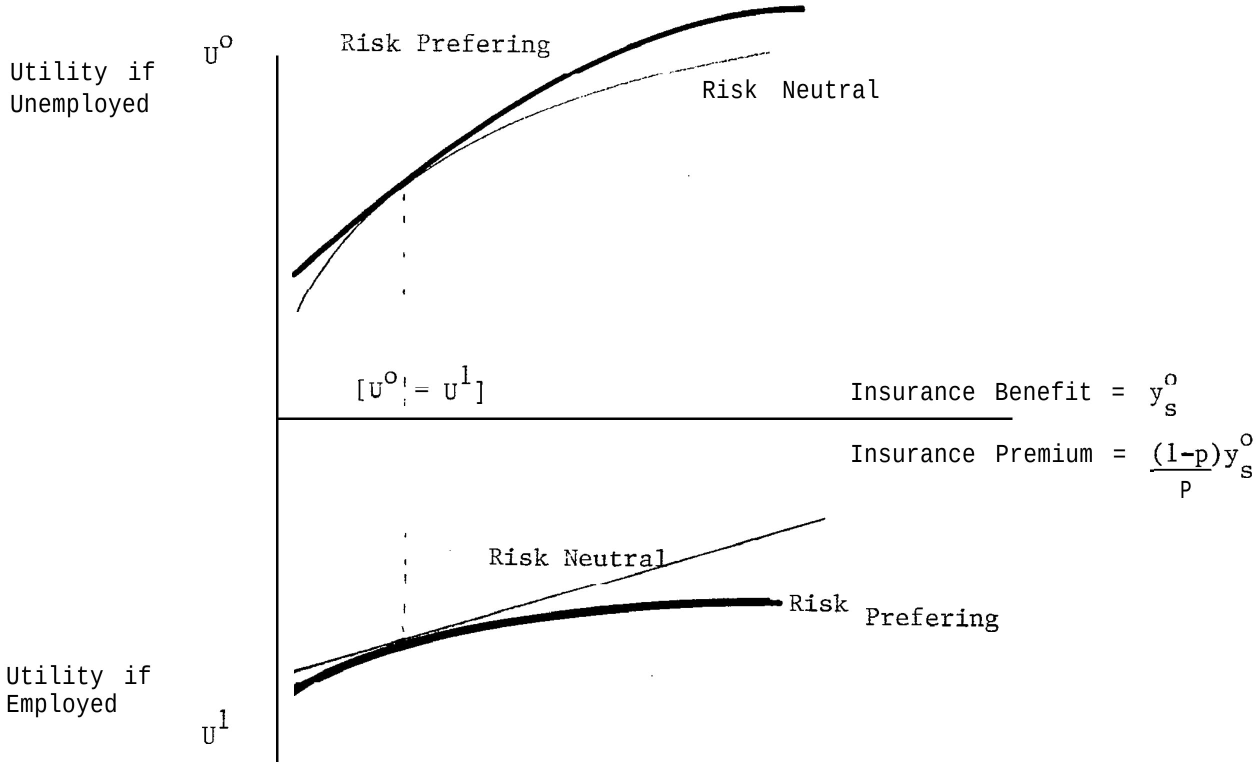
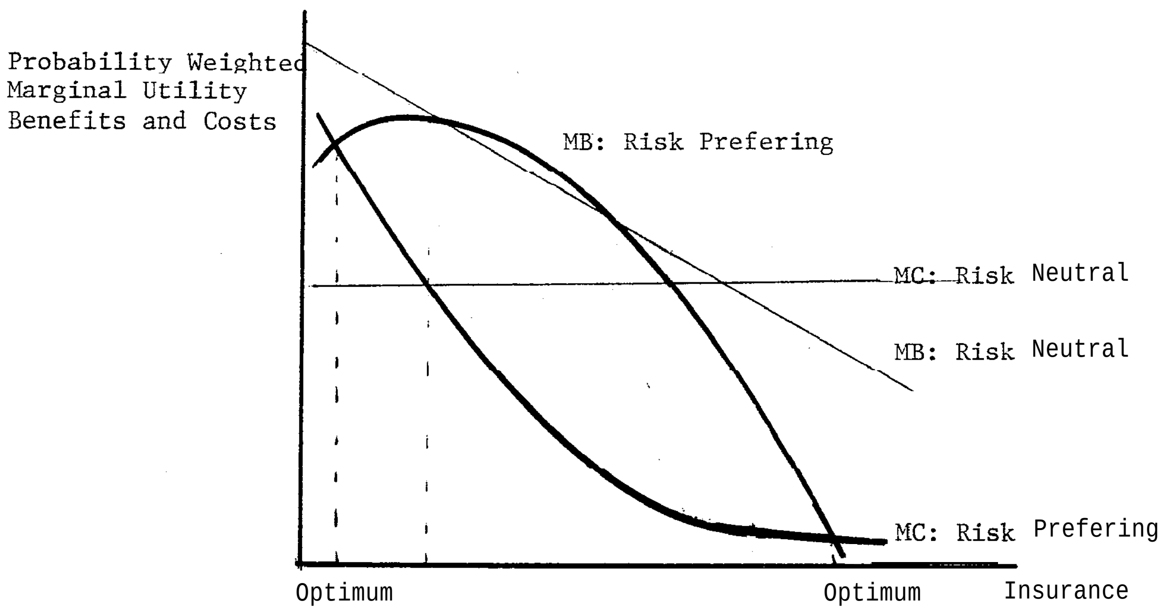


Figure 4

RISK PREFERING UTILITY FUNCTION



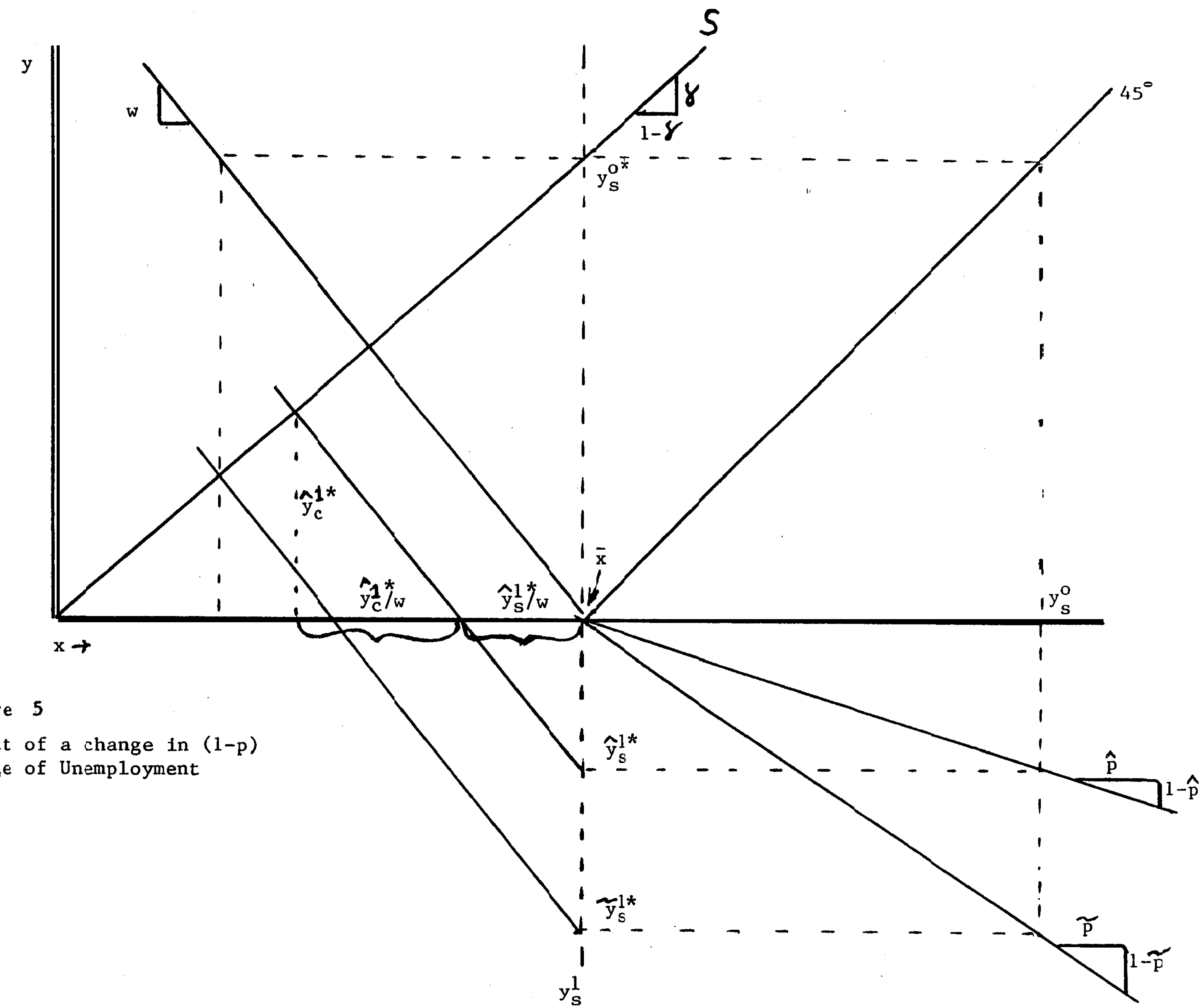


Figure 5

Effect of a change in (1-p)
Chance of Unemployment

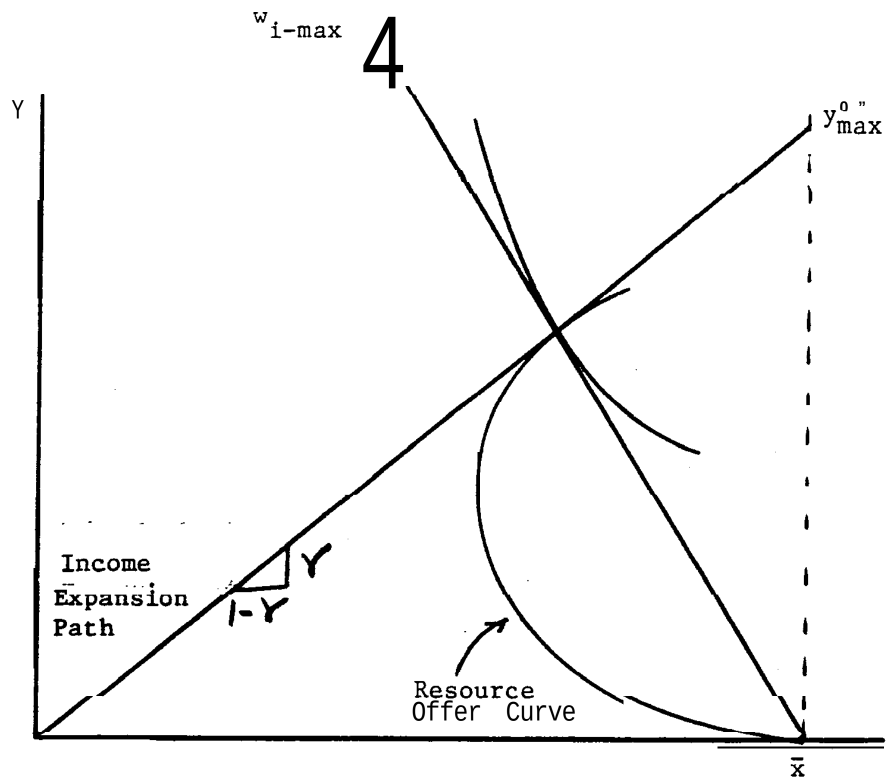


Figure 6

Wage Rate Which Induces
Maximum Insurance